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Discrete-Continuous Design Optimization of Steel Structures Using Simulated Annealing

Murat Nalçacı and Fuat Erbatur
Department of Civil Engineering, Middle East Technical University, 06531 Ankara, Turkey
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This paper is concerned with the use of the simulated annealing algorithm in optimum design of structures. Specifically, discrete and continuous size optimization of steel trusses is discussed. The objective function is chosen as the weight of the structure and stress, displacement and stability constraints are considered as per Turkish and AISC specifications. Numerical examples are solved using the computer program SAOPT developed for the purpose. Comparison of results obtained with previously published work reveals the efficiency of the SA approach in both continuous and discrete structural optimization problems.

Keywords: Structural optimization, simulated annealing, size optimum design, SAOPT.

1. Introduction

One major component of structural optimization is the optimization technique itself. During the last few decades interest in structural optimization has resulted in the integration of a variety of optimization techniques in optimum structural design problems. Simulated annealing, an artificial intelligence method is one of such techniques. It was first introduced by Kirkpatrick *et al.* [1] and independently by Cerny [2]. The method originates from the analogy between the simulation of the physical annealing of solids and problems of solving large scale combinatorial optimization problems. As for optimum structural design applications using SA, size optimization of planar and space trusses dominate the literature, although topology optimization and other structures, such as frames and plates are also considered. Some notable works are given in [3-11]. In this paper continuous and discrete size optimization of planar and space steel trusses is studied. The objective function is the weight of the truss. Stress, displacement and stability constraints as given in the Turkish and AISC specifications are used. For numerical applications a computer program (SAOPT) is designed [12]. SAOPT is composed of two parts; an analysis routine and an optimization routine. It is possible to integrate any commercially available analysis package to the program by introducing minor changes. Two numerical examples are given, both taken from the existing literature for comparison purposes.

2. Continuous-Discrete Size Optimization of Trusses

The continuous-discrete size optimization of trusses where the weight of the structure is taken as the objective function and the cross-sectional areas as design variables is posed as follows, Equations (1)-(5):

Find $\quad \mathrm{A} = [\mathrm{A}_1, \mathrm{A}_2, \ldots, \mathrm{A}_n]$ which minimizes

$$W = \sum \rho L_i Ai \tag{1}$$

subject to

- stress constraints:

$$\sigma_1 \leq \sigma_i \leq \sigma_u \tag{2}$$

- displacement constraints:

$$u_1 \leq u_k \leq u_u \tag{3}$$

- stability constraints:

$$\lambda_i \leq \lambda_{all} \tag{4}$$

- side constraints:

$$A_1 \leq A_m \leq A_u \tag{5}$$

where A_i are the design variables, W the objective function, ρ density of truss material, L_i length of member i and λ_i are buckling ratio for of member i. The subscripts l and u stand for the lower bound and upper bound values of the concerned quantities.

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- set the initial temperature so that the system is melted
  - randomly (not a must) generate the initial configuration
    - while termination criterion is not satisfied (i.e., while the system is not frozen)
      - while the inner loop criterion is not satisfied (i.e., while the equilibrium
        is not reached)
            * generate a new configuration Sj perturbing the current configuration Si
            * call accept (Si, Sj)
            * if [accept(Si, Sj)] then Si=Sj
       - end while
       - update the temperature T
     - end while
    - end while
    - return the optimum configuration
    - terminate the program
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Figure 1. Basic simulated annealing algorithm

3. AI Techniques and SA

Calculus-based techniques utilize gradient information and thus result in locally optimum solutions if the objective function is not smooth, unimodal and convex. Furthermore, these techniques deal with continuous variables rather than discrete variables. However, in many engineering optimization problems the design variables (cross-sectional areas of the truss elements) are discrete. For such problems two improved stochastic search techniques; simulated annealing and genetic algorithms, have emerged to be very promising. The common characteristics of these techniques are that they imitate the optimization process found in nature and that they fall into the class of artificial intelligence techniques. Simulated annealing is based on the physical annealing process of solids in heat bath. It is very effective in combinatorial optimization problems. It permits limited acceptance for higher values of the objective function and thus overcomes the difficulty to get trapped in poor local optima. Arora [13] lists, robustness, generality, accuracy, ease of use and efficiency as attributes of a good optimization algorithm. The experience gained in this study has shown that simulated annealing is quite a good algorithm. The basic simulated annealing algorithm is given in Fig. 1. The algorithm begins by generating an initial configuration. This initial configuration is an arbitrary configuration usually chosen randomly. At each step with constant T a new configuration S_j is randomly generated from the set of neighbouring configurations of the current solution S_i. The new configuration is accepted if its cost is less or equal to the cost of the current configuration. If the cost of the new configuration is higher, however, a probabilistic acceptance is applied.

4. Computer Applications - SAOPT Program

SAOPT (Simulated Annealing Based Structural Design and Optimization) program is developed for optimization of truss structures having a fixed geometry. The design variables are chosen as the sizes of the members. The analysis routine of the program is restricted to solve only three-dimensional truss structures to avoid extra variables, which may cause unnecessary complexity. SAOPT is designed for real life problems. The user can select either the Turkish or AISC specification as reference for the design. Constraints other than stress, stability and displacement constraints can very easily be inserted to the program and some of the existing constraints can be excluded from the program. For discrete vari-

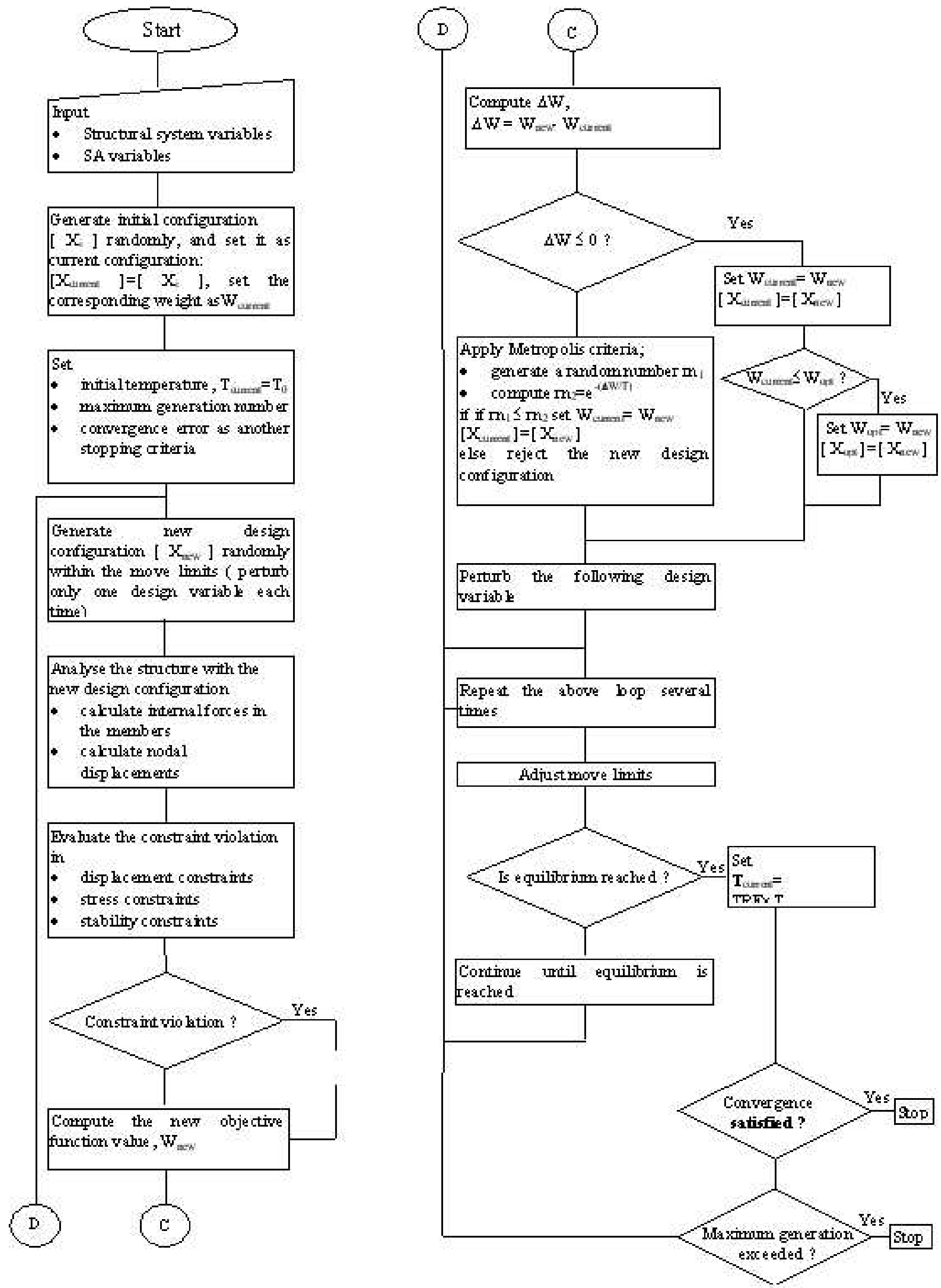

Figure 2. General flowchart of SAOPT

Table 1
Description of 25-bar truss problem

Test Problem: 25-bar space truss								
# of members: 25			# of joints: 6					
# of design variables: 16			# of loading conditions: 1					
MATERIAL PROPERTIES:								
Modulus of elasticity: 10^4 ksi								
Density of the material: 0.1 lb/in^3								
DISCRETE ELEMENT SET (in^2) (30 data):								
.1 .2 .3 .4 .5 .6 .7 .8 .9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2 2.3 2.4 2.5 2.6 2.8 3.0 3.2 3.4								
	Joint No		F_x(kips)		F_y(kips)		F_z(kips)	
LOADING	1		1.0		10.0		-10.0	
	2		0.0		10.0		-10.0	
	3		0.5		0.0		0.0	
	6		0.6		0.0		0.0	
CONSTRAINTS	Displacement Constraints: $\Delta_i \leq 0.35$ inch in x and y directions of nodes i=1,2 Stress Constraints: $-40ksi \leq \sigma_i \leq 40ksi \quad i = 1, ..., 25$ in x and y directions of nodes i=1.2							
MEMBER LINKING DETAIL:								
Group no	1	2	3	4	5	6	7	8
Members	A_1	$A_2 - A_5$	$A_6 - A_9$	$A_{10} - A_{11}$	$A_{12} - A_{13}$	$A_{14} - A_{17}$	$A_{18} - A_{21}$	$A_{22} - A_{25}$

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Table 2
Comparison of results of 25-bar truss

METHOD	WEIGHT (lb)	DESIGN VARIABLES (in^2)						
		A_1	$A_2 - A_5$	$A_6 - A_9$	$A_{10} - A_{11}$	$A_{12} - A_{13}$	$A_{14} - A_{17}$	$A_{18} - A_{21}$	$A_{22} - A_{25}$
Ref. 14	546.01	0.10	1.80	2.30	0.20	0.10	0.80	1.80	3.00
Ref. 4	481.33	0.10	0.40	3.40	0.10	2.00	1.00	0.40	3.40
Ref. 15	562.93	0.10	1.90	2.60	0.10	0.10	1.80	2.10	2.60
SAOPT	**484.33**	**0.10**	**0.40**	**3.40**	**0.10**	**2.20**	**1.00**	**0.40**	**3.40**
SAOPT (Cont.)	**464.459**	**0.0103**	**0.0393**	**3.6338**	**0.0100**	**1.9892**	**0.7770**	**0.1585**	**3.9210**

Table 3
Description of 72-bar transmission tower problem

	Test Problem: 72-bar transmision tower				
#of members: 72		#of joints: 20			
#of design variables: 16		#of loading conditions: 2			
MATERIAL PROPERTIES:					
Modulus of elasticity:10^4ksi					
Density of the material: 0.1 lb/in^3					
	Loading Condition	Joint No	F_x(kips)	F_y(kips)	F_z(kips)
LOADING	1	1	5	0	-5
	2	1	0	0	-5
	2	2	0	0	-5
	2	3	0	0	-5
	2	4	0	0	-5
CONSTRAINTS	Displacement Constraints: $\Delta_i \leq$0.25 inch in x and y directions i=1,...,20 Stress Constraints: -25 ksi $\leq \sigma_i \leq$ 25 ksi i=1,...,72				

MEMBER LINKING DETAIL:

Group No	Members	Group No	Members
1	$A_1 - A_4$	9	$A_{37} - A_{40}$
2	$A_5 - A_{12}$	10	$A_{41} - A_{48}$
3	$A_{13} - A_{16}$	11	$A_{49} - A_{52}$
4	$A_{17} - A_{18}$	12	$A_{53} - A_{54}$
5	$A_{19} - A_{22}$	13	$A_{55} - A_{58}$
6	$A_{23} - A_{30}$	14	$A_{59} - A_{66}$
7	$A_{31} - A_{34}$	15	$A_{67} - A_{70}$
8	$A_{35} - A_{36}$	16	$A_{71} - A_{72}$

Table 4
Comparison of results of 72-bar transmission tower

Design Variables (in^2)	Ref. 16	Ref. 17	Ref. 18	Ref. 19	Ref. 20	**SAOPT**
$A_1 - A_4$	0.161	0.1641	0.1565	0.1585	0.157	**0.1561**
$A_5 - A_{12}$	0.557	0.5552	0.5458	0.5936	0.537	**0.5561**
$A_{13} - A_{16}$	0.377	0.4187	0.4105	0.3414	0.411	**0.4180**
$A_{17} - A_{18}$	0.506	0.5758	0.5699	0.6076	0.571	**0.5639**
$A_{19} - A_{22}$	0.611	0.5327	0.5233	0.2643	0.509	**0.5264**
$A_{23} - A_{30}$	0.532	0.5256	0.5173	0.5480	0.522	**0.5097**
$A_{31} - A_{34}$	0.100	0.1000	0.1000	0.1000	0.100	**0.1000**
$A_{35} - A_{36}$	0.100	0.1000	0.1000	0.1509	0.100	**0.1000**
$A_{37} - A_{40}$	1.246	1.2893	1.2670	1.1067	1.286	**1.2238**
$A_{41} - A_{48}$	0.524	0.5201	0.5118	0.5793	0.516	**0.5046**
$A_{49} - A_{52}$	0.100	0.1000	0.1000	0.1000	0.100	**0.1000**
$A_{53} - A_{54}$	0.100	0.1000	0.1000	0.1000	0.100	**0.1000**
$A_{55} - A_{58}$	1.818	1.9173	1.8850	2.0784	1.905	**1.8717**
$A_{59} - A_{66}$	0.524	0.5207	0.5125	0.5034	0.518	**0.5290**
$A_{67} - A_{70}$	0.100	0.1000	0.1000	0.1000	0.100	**0.1000**
$A_{71} - A_{72}$	0.100	0.1000	0.1000	0.1000	0.100	**0.1000**
Weight (lb)	381.2	379.66	379.64	388.63	380.84	**379.2949**

able optimization, the user can select ready section files in the AISC and Turkish specifications or can create his own file. He can use different section files for different member types. The user can specify more than one loading condition as in practice. The flowchart given in Fig. 2 summarizes the steps in a typical execution of SAOPT. For illustration of the SA algorithm two example problems deliberately chosen from the literature to allow comparison with other methods are given below.

4.1. 25-Bar Space Truss

This illustrative example deals with the optimum design of 25-bar space truss shown in Fig. 2. The structural members are made of aluminum with a modulus of elasticity of E = 10^4 ksi and a density of 0.1 lb/in^3. (US units are deliberately used to allow easy comparison with available literature results. Note that, 1 in.= 2.5cm, 1 kip=4.53 kN, 1 lb=0.453 kg.) The truss is subjected to single loading condition as described in Table 1.

The objective function is the weight of the truss. The design variables are the cross-sectional areas of the members and are chosen from a discrete element set consisting of 30 members. The number of design variables is reduced to 8 through member linking as shown in Table 1. The design space consists of 30^8 possible design configurations. The stresses in the members are limited to ± 40 ksi and a maximum of 0.35 inch displacement is allowed in x and y directions at nodes 1 and 2. Although the original problem is an example of discrete optimization it is also solved with continuous variables to illustrate the effectiveness of SA in continuous optimization. The comparison of the results, Table 2, indicates the superiority of SA based approaches to other methods.

4.2. 72-Bar Transmission Tower

The second illustrative example is the 72-bar transmission tower, Fig. 3, a frequently seen example of continuous optimization. The design objective is the minimum weight of the structure. Member linking is performed and the problem is reduced to have sixteen independent variables, Table 3. Two loading conditions are imposed to thestructure as shown in Table 3. A maximum of ± 25 ksi stress is allowed in the members and the displacements at the nodes are limited to 0.25 inches in both x and y directions. The results obtained are given in Table 4 together with results of previously published literature for comparison purposes.

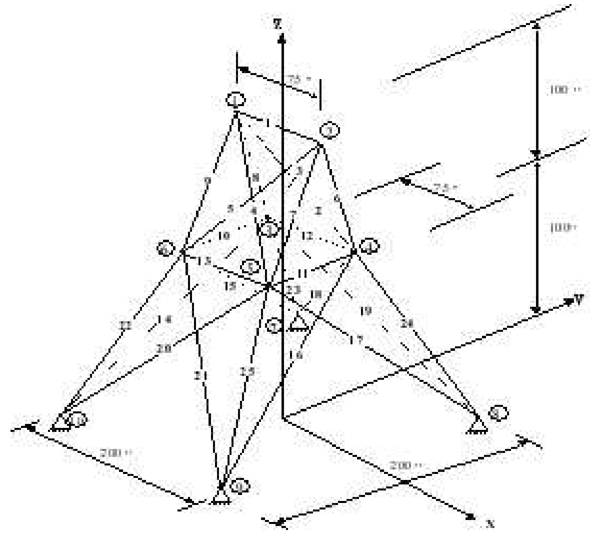

Figure 3. 25-bar space truss

5. Remarks and Conclusions

The search for the optimum structural system under prescribed conditions has always been the primary objective of structural engineers. As a result of this naturally attractive goal, structural optimization has become one of the major fields in engineering. Its applicability in a wide range of practical design problems attracts great interest and thus leads to a continuing motivation for research. Practical design problems require a lot of design variables and constraints and their solutions contain many local minima. As a result of these factors the computational expense increases which constitute the main obstacle in the optimization of structural systems. Thus, computer algorithms must handle these large-scale problems reliably and efficiently. The structural design process is generally characterized by discrete design variables, which are chosen from a predetermined set. On the other hand, most of the optimization techniques only handle continuous optimization and thus not satisfactory in the broad spectrum confronting a design engineer. This leads to the development of new techniques, which can deal with discrete optimization prob-

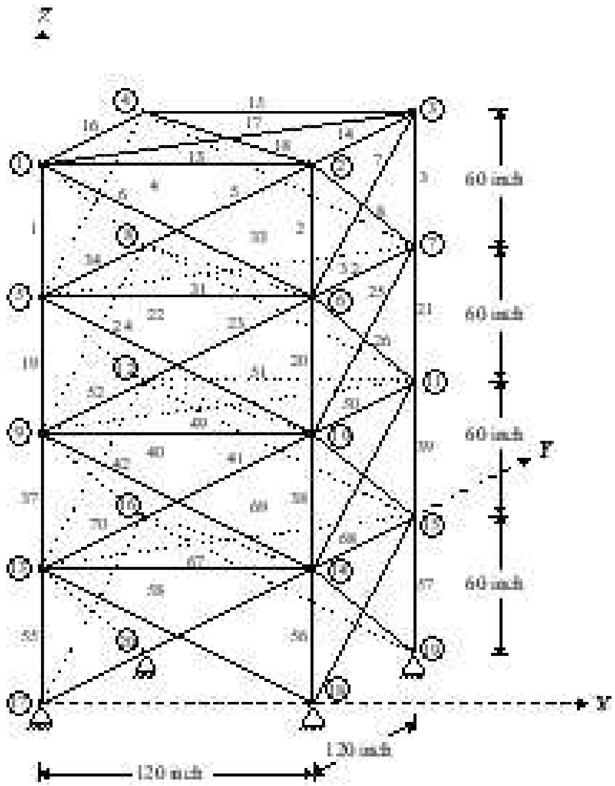

Figure 4. 72-bar transmission tower

lems. Artificial intelligence techniques are such techniques and simulated annealing is a promising representative.

In this paper the use of simulated annealing heuristic for the size optimization of planar and space trusses is investigated. For this purpose the computer program SAOPT is developed to meet the requirements of both the practicing engineers and researchers. A variety of example problems are solved. Two of them are reproduced here for illustrative purposes. All examples are deliberately chosen from the literature to allow a comparison of results. Both continuous and discrete optimizations are performed. The results obtained are promising revealing that the SA technique can effectively be used in the optimum design of structural members and systems.

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