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A New, CFA-Based Proportional-Integral-Derivative Controller

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The basic aim of this paper is to present a new controller circuit comprising a commercially available current-mode active component. This active element is called as current-feedback amplifier (CFA) and exhibits excellent properties compared to their voltage-mode counterparts, op-amps. The active element used in this work is AD844 from Analog Devices. The use of this active component yields simple implementations and faster controllers. The proposed PID circuit is applied to a third-order attitude control system to verify the given specifications.

Keywords: Proportional-integral-derivative controller, current-feedback amplifier, signal-flow graph.

1. Introduction

The proportional-integral-derivative (PID) controllers are one of the most important control elements used in process control industry. In practice, operational amplifiers are generally used in analogue controllers. However, op-amps suffer from low bandwidth and poor slew-rate performances. On the other hand, the AD844 is a commercially available versatile active component providing an excellent AC and DC performance like other commercially available CFAs. It combines high bandwidth and very fast large signal response with excellent DC performance. It is also free from the slew rate limitations inherent in traditionally op-amps and other current-feedback op-amps. It can be used instead of traditional op-amps, however its current feedback architecture results in much better AC performance and high linearity [1]. CFA is equivalent to the combination of a second-generation current-conveyor having a positive unity gain (CCII+) and a unity gain voltage buffer [2]. Therefore, this active element can be considered as a collection of unity gain elements, which are high performance active basic blocks.

In spite of the above-mentioned features, no other work has been carried out for the generation controllers using current feedback amplifiers except the ones by Erdal et. al. [3-5]. The main purposes of this paper is to present a new circuit for the realisation of PID controller using only four CFAs and passive components. In this new PID circuit, the PID coefficients are independent from voltage tracking behaviour of the y-x inputs of the CFAs comparing with the previous work [4]. The SPICE simulations of the proposed circuit are fulfilled to verify the theoretical predictions. Also the proposed PID circuit is applied to a third-order attitude control system to verify the given specifications.

2. Current-Feedback Amplifier (CFA)

The circuit symbol and its equivalent circuit of a current feedback amplifier (CFA) are shown in Fig. 1.

An ideal CFA can be defined by the following equations [2]:

$$V_x = V_y, \quad V_o = V_z, \quad I_y = 0, \quad I_z = I_x. \tag{1}$$

An equivalent circuit of CFA is also shown in Fig. 1. Ideally a CFA has a zero x-input resistance, whereas the y and z-terminal resistances are infinite and the capacitances of these terminals are zero. In practice however, these values can be obtained as with nonzero and finite appropriate amounts. As example for AD844 from Analog Devices, $R_x = 50\Omega$, $R_z = 3M\Omega$ and $C_z = 4.5pF$ are the typical values [1].

Note that both plus and minus signs or the letters y and x are used in literature to denote the inputs of CFA. For example see the references [6,7]. In this study, corresponding to the current conveyor terminology, y and x are preferred for the inputs of the commercially available current feedback amplifier, AD844.

Taking also the active element non-idealities into account, the terminal equations of CFA can

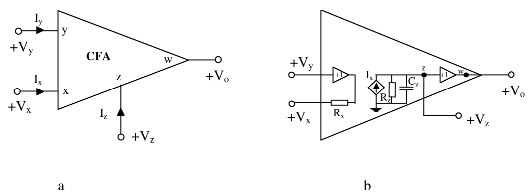

Figure 1. a) Circuit symbol of CFA b) Equivalent circuit of AD844, a commercially available CFA

be rewritten as follows:

$$V_x = \beta V_y, \quad V_o = \gamma V_z, \quad I_y = 0, \quad I_z = \alpha I_x \quad (2)$$

Here $\alpha = 1 - \varepsilon_i$ and $\beta = 1 - \varepsilon_v$ denote the current and voltage gain of the current conveyor, and $\gamma = 1 - \varepsilon_o$ denotes the voltage gain of the voltage buffer, where ε_i, ($|\varepsilon_i| << 1$), ε_v, ($|\varepsilon_v| << 1$), and ε_o, ($|\varepsilon_o| << 1$) are the current tracking error, the voltage tracking error of the input buffer and the voltage tracking error of the output buffer, respectively. Furthermore, the low output impedance of the buffer enables easy cascading in voltage-mode operation.

3. Synthesis Procodure

The transfer function of a general analog, PID controller can be written as follows [8]:

$$T(s) = \frac{V_0(s)}{V_i(s)} = K_P + \frac{K_I}{s} + sK_D = \frac{K_D s^2 + K_P s + K_I}{s}. \quad (3)$$

One of the signal-flow graph models realising the transfer function is shown in Fig. 2, assuming that the node signals are voltages. It is not explained here how to obtain this graph, but those interested may easily verify using the Mason gain formula, that the graph transfer function from the input node i to the output node o is equal to T(s) in Eq. (3) [8].

Using this signal-flow graph, the controller transfer function T(s) can be realised using the active-RC circuits involving AD844s [9]. For this realisation, it is sufficient to know the active sub-circuits corresponding to the sub-graphs between V_i, and V_o. These sub-graphs and their corresponding active sub-circuits including AD844s, are shown in Fig. 3.

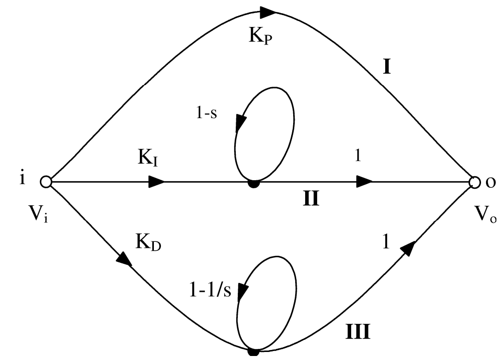

Figure 2. A signal flow graph corresponding to the transfer function of the proportional-integral-derivative (PID) controller

In Fig. 3(a), an amplifier circuit and its signal-flow graph are shown. The gain of this amplifier circuit is $-R_2/2R_1$. In Fig. 3(b), an integration circuit and its signal-flow graph are shown. The integration time constant of this circuit is $-1/2R_3C_1$ as it can be easily seen from the signal graph. In Fig. 3(c), a CFA based derivative circuit and its signal-flow graph is shown. The derivation time constant of this circuit is $-R_4C_2/2$ as it can be easily seen from the signal graph. The proposed three mathematical functions, i.e. multiplication, integration, and derivation, are transmitted to the output by the CFA based summing circuit shown in Fig. 3(d).

If a given transfer function is represented by a signal-flow graph composed of sub-graphs as shown in Fig. 2, then the circuit corresponding to the signal-flow graph can be realised by interconnecting the building blocks of Fig. 3. The PID controller circuit realising the transfer function T(s) can easily be obtained by interconnecting the sub-circuits of Fig. 3 according to the overall signal-flow graph in Fig. 2 as follows in Fig. 4:

The realisation of the analogue CFA-based, PID controller circuit corresponding to the signal-flow graph in Fig. 2, which is realised by using the sub-circuits given in Fig. 3 is illustrated in Fig. 4. Note that in Fig. 4, removing the path III and path II can obtain a PI controller circuit and a PD controller circuit respectively.

If the circuit in Fig. 4 is analysed with taking the non-idealities of CFA into account the control coefficients K_P, K_I, and K_D will be obtained as

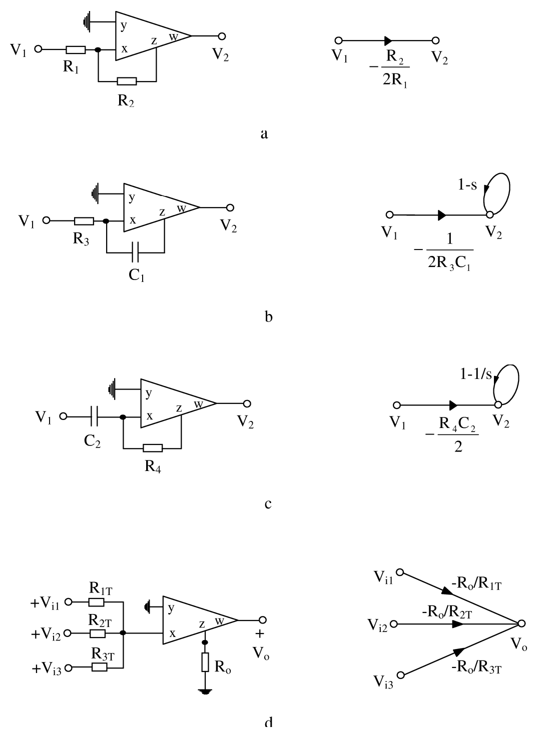

Figure 3. Sub-graphs and corresponding active sub-circuits involve AD844s.

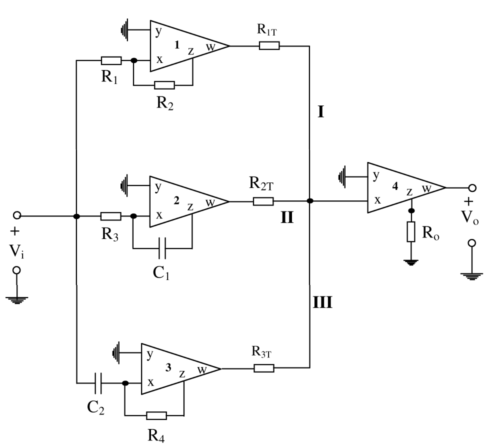

Figure 4. Proposed CFA-based PID controller realisation corresponding to the signal-flow graph shown in Fig.2.

follows:

$$K_P = \frac{\alpha_1\gamma_1\alpha_4\gamma_4}{(1+\alpha_1)}\frac{R_2R_0}{R_1R_{1T}}, \tag{4}$$

$$K_I = \frac{\alpha_2\gamma_2\alpha_4\gamma_4}{(1+\alpha_2)}\frac{R_0}{R_3C_1R_{2T}}, \tag{5}$$

$$K_D = \frac{\alpha_3\gamma_3\alpha_4\gamma_4}{(1+\alpha_3)}\frac{R_4C_2R_0}{R_{3T}}. \tag{6}$$

It should be noted that the Eqns. (4,5,6) are independent from the voltage gain of the output buffers (β_i, i=1,...,4) because all the y inputs are connected to ground. In another words, the sensitivities of PID coefficients with respect to the voltage gains of CFAs are zero. It should be noted that the sensitivities of the PID coefficients with respect to the other active and passive element parameters are no more than unity.

If the ideal case is considered, i.e.,

$$\alpha_i = 1, \;\; i = 1...4, \;\; \gamma_k = 1, \;\; k = 1...4, \tag{7}$$

the control coefficients will be given as follows:

$$K_P = \frac{R_2R_0}{2R_1R_{1T}}, \tag{8}$$

$$K_I = \frac{R_0}{2R_3C_1R_{2T}}, \tag{9}$$

$$K_D = \frac{R_4C_2R_0}{2R_{3T}}. \tag{10}$$

Note that the appropriate resistors can adjust K_P, K_I, and K_D independently, and also the total number of CFAs used in this design is only four, which is minimum. Since in a general PID function there exist three different coefficients we need at least three CFAs and one more for summing operation in order to realise these arbitrary coefficients.

4. Simulation Results

In order to confirm the theoretical results, the PID circuit given in Fig. 4 is simulated with the SPICE program by using the macromodel of

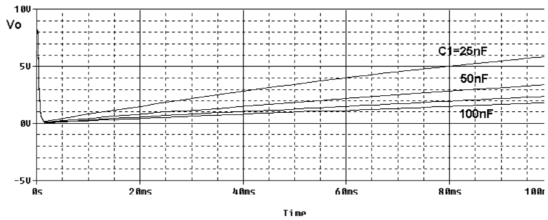

Figure 5. Results of the simulation of the PID controller with R_1=10MΩ, R_2=0.1Ω, R_{1T}=R_{2T}=R_{3T}= Ro=10kΩ C_2=10μF, R_3=2.5kΩ, R_4=100kΩ, for (C_1=25nF, 50nF, 75nF, 100nF)

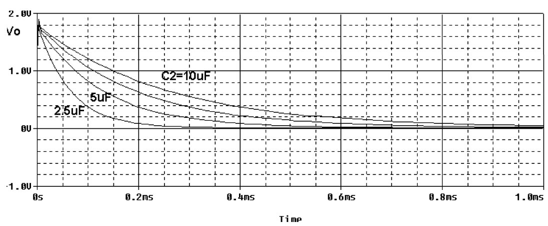

Figure 6. Results of the simulation of the PID controller with R_1=10MΩ, R_2=0.1Ω, R_{1T}=R_{2T}=R_{3T}= Ro=10kΩ C_1=10μF, R_3=2.5kΩ, R_4=100kΩ, for (C_2=2.5μF, 5μF, 7.5μF, 10μF)

AD844/AD from Analog Devices. In this circuit, supply voltages of ±12V are used. The values of the capacitor C_1 are varied as 25nF, 50nF, 75nF, and 100nF when the peak value of the input voltage was kept 10mV with a pulse duration time T_{pulse}=100ms. The values of the capacitor C_2 are varied in the range from 2.5μF to 10μF with the 2.5μF increments when the peak value of the input voltage was kept 1mV with a pulse duration time T_{pulse}=100ms. The simulation results of the output of the CFA-based PID controller are given in Figure 5 and 6 respectively. In both situations, the proportional gains are taken to be K_P=10^{-8}, i.e. $R_2/R_1 << 1$. The values of the capacitors and resistors in simulation procedure are given in figure caption. The capacitance values of the integrator are selected especially smaller. In this way, the properties of this circuit can be obtained in a short simulation process. From Figs. 5 and 6 it is clear that the results are in good agreement with the theoretical expectations.

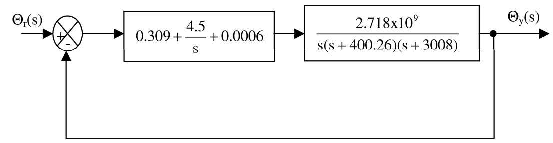

Figure 7. Block diagram of the third-order closed-loop attitude control system with PID control

In the following, the proposed CFA-based PID circuit will be applied to the third-order airship attitude control system represented by the forward-path transfer function given as [8].

$$G_P(s) = \frac{\Theta_y(s)}{\Theta_e(s)} = \frac{2.718 \times 10^9}{s(s+400.26)(s+3008)}. \quad (11)$$

Let us take the parameters of the PID controller to satisfy the time-domain performance specifications given in the reference [8], such as follows:

$$K_P = 0.309, K_I = 4.5s^{-1}, K_D = 0.0006s. \quad (12)$$

Then the closed-loop control system will be given in Fig. 7 and the closed-loop transfer function becomes as follows:

$$G(s) = \Theta_y(s)/\Theta_r(s) =$$

$$2.718x10^9 \times (0.309 + 4.5/s + 0.0006s)/$$

$$s(s+400.26)(s+3008) + 2.718 \times 10^9$$

$$\times(0.309 + 4.5/s + 0.0006s). \quad (13)$$

Figure 8 shows the unit-step response of the third-order closed-loop attitude control system with the proposed,CFA-based PID controller. Notice that the unit-step response of the third-order closed-loop attitude control system with the proposed, CFA-based PID controller has the same properties with the given example in the reference [8]. This proves that the proposed, CFA-based PID circuit works very well.

5. Conclusions

In this study, a new alternative to implement a CFA-based PID controller has been proposed.

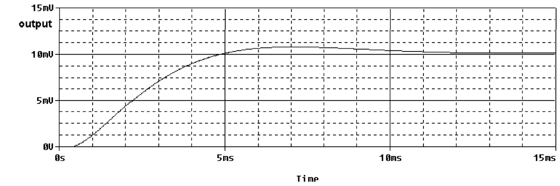

Figure 8. Step input response of the third-order closed-loop attitude control system with PID control

The proposed PID circuit has an advantage over the previous one that was presented by the author [4]. The new circuit result in the PID coefficients independent from the voltage tracking behaviour of the y-x inputs of the CFAs. The method presented here is straightforward and simple, as it provides not only the configuration but also the element values directly from the coefficients of the PID controller voltage transfer function. The use of the commercially available active component (i.e. AD844s) simplifies implementation and also makes possible to control the rapidly changing signals. Selecting the impedance-scaling factor properly as stated by Svoboda [7] could reduce the effects of parasitic input impedance of the AD844 on controller performance. Since the AD844 is equivalent to the CCII+ buffered with a unity gain voltage amplifier, the proposed circuit is comparable with the controller circuit given in [3].

Finally, another AD844 controller configuration different from that proposed in this paper can be derived in a similar way using a different signal-flow graph model.

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