

Conformal Symmetry and Triviality

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We study examples where conformal invariance implies triviality of the underlying quantum field theory.

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1. Introduction

Having conformal symmetry in a quantum field theoretical model implies that the Lagrangian of the model does not possess any dimensional parameters. The mass term or any dimensional coupling constant is sufficient to break this symmetry. Since any cut-off term also means a dimensional parameter, the model should be free from infinities. It should not be necessary to renormalize the theory in any fashion.

Here we will study the implications of conformal symmetry in quantum field theory and try to cite examples where imposing conformal symmetry results in triviality.

2. Examples

Imposing conformal symmetry on a model means the absence of any dimensional parameter in the theory, whether this parameter is present in the original lagrangian, or it is dragged into the theory via the renormalization process. The fulfillment of these restrictions still does not guarantee that the model has conformal symmetry. Unless the space-time dimensions equals two, we have to make sure that the two point Wightman function of the model should be [1] an integer power of

$$(x - y)^2 + i\epsilon(x_0 - y_0)$$

In two dimensions, this function is given as logarithm of $(x - y)^2 + i\epsilon(x_0 - y_0)$ for scalar fields. If we use a non-integer power of this term [2] the index comes out of the log function as a coefficient, still keeping the same form. In higher dimensions a power of the same function have to be used. If this power is not an integer, we can change the

sign between the two terms in $(x - y)^2 + i\epsilon(x_0 - y_0)$ by a conformal transformation. If the power of this expression is an integer, we can separate this term into its real (Principal value) and imaginary (delta function) parts and show that under conformal transformations both terms transform covariantly, keeping the sign as it was before the transformation. If the power is not an integer, this separation is not possible. In all these remarks we wanted to maintain conformal symmetry in the Wightman function sense.

It was shown [3] that if this power is a rational number, you can carry the local conformal symmetry to the global one in the covering space in the general sense. If the power is an irrational number, even this is not possible by using a countable number of sheets in the covering space.

In 1 + 1 dimensions spin is a parameter which can take any value. The presence of conformal symmetry was shown to be present in Thirring model for discrete values of spin and coupling constant [4]. Similar behaviour is also seen in the analysis of the conformal group in 1 + 1 dimensions if one uses "analytical representations" [5].

The rational indices in physics also occur in statistical mechanical models when conformal symmetry is imposed. It is found that [6] the indices used in phase transitions should be discrete and rational if we want to satisfy the unitarity constraint. One can not extend this construction to relativistic quantum field theoretical models, though, and show the existence of non-trivial models in this field. The question is whether the rationality of the different powers (indices) that describes these models also makes these models trivial. Much work was done when the central charge c is equal to unity. I do not recall work when $c > 1$ in relativistic quantum field theoretic

cal models using local fields, but this may be only due to my ignorance on this field.

Schroer [7] remarks that if a model is built in Minkowski space, imposing conformal symmetry on the model may result in making the model "trivial". He also points out to the fact that in anti-de Sitter (AdS) spaces, however, the Hamiltonian may be replaced by a generator of the symmetry, which may make a non-trivial construction possible. He also remarks that this construction seems improbable [8].

The connection between AdS spaces and conformal symmetry was shown in the early nineties [9]. Maldacena made the subject popular a little later [10]. As far as I know it is an open problem whether one can construct non-trivial generic models higher than two Lorentzian dimensions with global conformal symmetry. We all know that the only example in $d = 4$, $N = 4$ super Yang-Mills theory exists. We want to find other examples.

By "triviality" we mean models where interactions vanish as the cut-off is removed. A famous example is the Baker-Kincaid paper on the ϕ^4 model in four dimensions [11]. Ken Wilson pointed out to similar phenomena in $d = 4$ earlier [12]. Klauder's recent work shows that this problem is not completely solved yet [13].

Additional references can be found in the thesis of Reenders [14], submitted to University of Groningen. Here it is shown how the Nambu-Jona-Lasinio model [15] goes to a trivial model, since the coupling constant is proportional to a negative power of the cut-off or the correlation length. It is essential that the space-time dimension is four or larger for this to occur. For $d = 3 + 1$, the coupling constant is proportional to a negative power of the logarithm of the cut-off. For higher dimensions, it is proportional to a negative power of the cut-off. As the cut-off or the correlation length goes to infinity, the interaction vanishes.

A related phenomenon was shown in quantum electrodynamics (QED) long ago [16]. Here one shows how the virtual fermion-antifermion pairs form dipoles which screen the charge.

Gell-Mann and Low [17] stated that to have a non-trivial theory, the derivative of the coupling constant with respect to the cutoff,

$$\beta = \Lambda \frac{dg}{d\Lambda}$$

should have a non-trivial fixed point. The two [18] and approximate three-loop [19] calculations of QED were consistent with the triviality of the model. Attempts to go beyond perturbation theory [20] did not bring any results that contradict this fact. The fact that QED results are confirmed by experiments in an excellent manner makes many forget that QED is actually an asymptotic model.

Additional work on his problem was done by many groups. This work is described in Reenders' thesis [14]. Bardeen and co-workers [21] showed that in the "quenched ladder" approximation of QED, the dimension of the four fermion operator is no longer six but four at a critical coupling. This fact makes it necessary to add Nambu-Jona-Lasinio type terms to the QED Lagrangian, since these terms have the same dimensions as the interaction of the model. Then the new model, named as gauged Nambu-Jona-Lasinio model, should replace QED for this range of values of the fermion-photon coupling constant. Groups led by Miransky [22], Appelquist [23] and Kogut [24] further studied this model and confirmed these results. Lattice calculations [25] also confirm that pure QED is trivial. Reenders studies the gauged Nambu-Jona-Lasinio model with internal symmetry using non-perturbative methods [26], and concludes that if there are more than 44 (in another approximation 54) flavors of fermions, scalar and pseudo-scalar bound-states made out of fermions will prevent the total screening of the charge, resulting in a non-trivial conformal invariant model.

Similar examples of "triviality" were studied in the eighties in fermionic models with some form of conformal symmetry [27]. These models are quantized using $\frac{1}{N}$ methods resulting in effective lagrangians as

$$L_{eff} = \bar{\psi} \gamma^\mu (\partial_\mu - m - e A_\mu) \psi.$$

The bosonic kinetic term is generated by the one-loop correction, which also shows that the coupling constant goes to zero as the cut-off is removed. The question is whether infinities in higher order calculations will cancel this zero, resulting in finite results for physical processes. Two and three loop calculations for several processes show [28] that indeed a finite result is possible, but it is exactly equal to the naive-quark result. One can not obtain the logarithmic corrections.

3. Conclusion

The examples studied above suggest that imposing conformal symmetry makes models "trivial". One may still add additional terms to these models, study the behaviour of their beta functions, and see if there are non-trivial fixed points of these models. This may not be easy to do, since the model should be studied in a non-perturbative manner, repeating at least partially the Johnson-Willey-Baker [20] calculations for this model. I still think this will be a worthwhile endeavor to follow up.

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