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Two Photon Physics in Very Peripheral Collisions of Relativistic Heavy Ions

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At relativistic heavy ion colliders, electromagnetic fields of heavy nuclei are very strong and can produce a wide variety of leptons. We present recent results from CERN and RHIC on ultra-peripheral interactions, focusing on electron-positron pair production.

Keywords: Relativistic collisions, heavy ions, lepton-pair production

1. Introduction

Particle production via electromagnetic processes in peripheral collisions of relativistic heavy ions has been studied recently [1-7] both experimentally and theoretically, because it is important at colliding beam accelerators. As an example, fully stripped heavy ions provided by relativistic Heavy-Ion collider (RHIC) and Large Hadron collider (LHC) will collide at 100 GeV and 3400 GeV per nucleon at the center of mass energies respectively. Main goal of these experiments are to create a so-called quark-gluon plasma and to study the physics associated with it. It is expected that ultra-relativistic heavy-ions can provide this form of matter in the central, or near-central collisions. In these central collisions, thermodynamic conditions may become sufficient to deconfine the constituent quarks and gluons of baryons and mesons into a short-lived plasma state. Lepton pairs, especially electron and muon-pair production, from hadronic interactions can help us to monitor the formation and decay of the quark-gluon plasma phase of matter. In these collisions, lepton-hadron final state interactions are generally small, therefore leptons may carry direct informations on the space-time region of their creation.

When heavy ions collide at relativistic velocities, the Lorentz-contracted electromagnetic fields in the space-time region near the collision are sufficiently intense to produce large numbers of electron-positron pairs, muon pairs, vector bosons, weak bosons (Z_0, W^+, W^-) or possibly the yet-unconfirmed Higgs bosons. These collisions are fundamentally different from those involving single charged projectiles because the strength of the coupling constant $Z\alpha$, where Z is the charge and α is the fine structure constant, can be large. Lepton pair production in these systems depends on energy and charge of the colliding nuclei. Although the first order perturbative calculation gives reliable answer for the low charge and energy, these calculations give some unphysical results for the higher charge and energies.

Due to the strong, long range electromagnetic forces between heavy-ions, which are peripheral in nature, single/multiple-lepton pair production from the electromagnetic fields of the heavy-ions is a major contribution to the physical background and several authors suggested that these lepton pairs may mask the leptonic signals coming from the quark-gluon plasma phase [8, 9] via the Drell-Yan process. There are also two dominant electromagnetic processes that can lead to beam depletion at RHIC. The electron capture process following pair production during heavy-ion collisions and the Coulomb disassociation of the heavy-ions, following the electromagnetic excitation of the giant dipole resonance are the dominant modes of beam loss mechanism at RHIC. This collider will provide ultra-relativistic fully stripped heavy-ion beams, and the above electromagnetic processes cannot be ignored in detector or accelerator design.

2. Formalism

In recent articles [10, 11], we have derived the lepton pair production cross section by using the second order (s.o.) (Fig. 1) perturbative method. The four potentials of one the colliding nuclei can

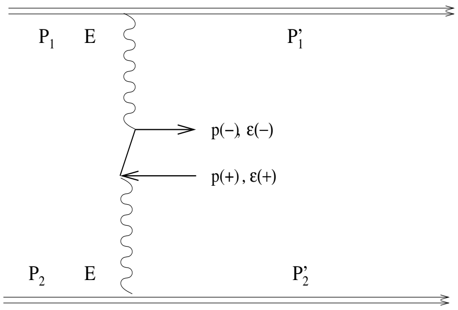

Figure 1. The lowest order Feynman diagrams for the electron-positron pair production. In addition to this direct term, there is also one exchange term.

be written as

$$\begin{aligned} A_o^{(A)}(\rho; q) &= -2\pi\, Ze\, \delta(q_o + \beta q_z)\, \gamma^2 \\ &\times \frac{e^{-i\mathbf{q}_\perp \cdot \rho/2}}{q_z^2 - \gamma^2 \mathbf{q}_\perp^2}\, G_E(q^2)\, f_z(q^2) \\ A_z^{(A)}(\rho; q) &= -\beta\, A_o^{(A)}(\rho; q)\ , \end{aligned} \tag{1}$$

where β is the velocity of the ion, $G_E(q^2)$ is the form factor of the nucleus, and $f_Z(q^2)$ is the form factor of the proton. The interaction for particle B, $A_\mu^{(B)}(\rho; q)$, is found from $A_\mu^{(A)}(\rho; q)$ by substituting $\beta \to -\beta$ and $\rho \to -\rho$. The S-matrix is given by

$$\begin{aligned} &\langle\, \mathbf{k}\, \sigma' \,|\, S_\rho \,|\, \mathbf{q}\, \sigma \,\rangle = -ie^2 \frac{m}{\sqrt{E_k E_q}} \\ &\times \int \frac{d^4p}{(2\pi)^4}\, \bar{u}^{\sigma'}(\mathbf{k}')\, \not{A}^{(A)}(\rho; k-p) \\ &\times \frac{\not{p} + m}{p^2 - m^2}\, \not{A}^{(B)}(\rho; p+q)\, v^{(\sigma)}(\mathbf{q})\ . \end{aligned} \tag{2}$$

To write this in an explicit form that we may calculate, we decompose the Feynman propagator into its positive and negative frequency parts,

$$\begin{aligned} \frac{\not{p} + m}{p^2 - m^2} &= \frac{1}{p_o - E_p} \frac{m}{E_p} \sum_\sigma u^{(\sigma)}(\mathbf{p})\, \bar{u}^{(\sigma)}(\mathbf{p}) \\ &+ \frac{1}{p_o + E_p} \frac{m}{E_p} \sum_\sigma v^{(\sigma)}(\mathbf{p})\, \bar{v}^{(\sigma)}(\mathbf{p}) \end{aligned} \tag{3}$$

We also change to non-relativistically normalized spinors

$$\begin{aligned} u^{(\sigma)}(\mathbf{p}) &= \sqrt{\frac{E_p}{m}}\, |\, u_\sigma^{(+)} \,\rangle \\ v^{(\sigma)}(-\mathbf{p}) &= \sqrt{\frac{E_p}{m}}\, |\, u_\sigma^{(-)} \,\rangle\ , \end{aligned} \tag{4}$$

where the norm on the spinors becomes $\langle\, u_{\sigma'}^{(\pm)} \,|\, u_\sigma^{(\pm)} \,\rangle = \delta_{\sigma'\sigma}$, and the change in sign on $\mathbf{p}$ is to be accompanied by $p_o = -E_p$ for the anti-particle. This is done to match the nuclear physics convention for labeling hole states. The interaction is also rewritten using

$$\begin{aligned} \gamma_o \gamma_\mu A^\mu(\rho; p) &= \alpha \cdot \mathbf{A}(\rho; p) - \beta A_o(\rho; p) \\ &= \alpha_z A_z(\rho; p) - \beta A_o(\rho; p) \end{aligned} \tag{5}$$

where β is the Dirac matrix β. For the direct term, the term which arises from the first term on the right of Eq. (3), we find

$$\begin{aligned} &\langle\, \mathbf{k}\, \sigma_k \,|\, S_\rho^d \,|\, \mathbf{q}\, \sigma_q \,\rangle = i \sum_s \sum_{\sigma_p} \int \frac{d^4p}{(2\pi)^4} \times \\ &\times\ \frac{A_o^{(A)}(\rho; k-p)\, A_o^{(B)}(\rho; p-q)}{E_p^{(s)} - p_o} \\ &\times\ \langle\, u_{\sigma_k}^{(+)} \,|\, (1 - \beta\alpha_z) \,|\, u_{\sigma_p}^{(s)} \rangle \\ &\times\ \langle\, u_{\sigma_p}^{(s')} \,|\, (1 + \beta\alpha_z) \,|\, u_{\sigma_q}^{(-)} \,\rangle\ , \end{aligned} \tag{6}$$

where β is the velocity of the ion. If one wants the total cross section, the integral over ρ can trivially be done to give

$$\begin{aligned} \sigma &= \frac{1}{4\beta^2} \sum_{\sigma_k \sigma_q} \int \frac{d^3k\, d^3q\, d^2p_\perp}{(2\pi)^8} \Big| \mathcal{S}^{(+)}(k, q : \mathbf{p}_\perp) \\ &+ \mathcal{S}^{(-)}(k, q : \mathbf{k}_\perp + \mathbf{q}_\perp - \mathbf{p}_\perp) \Big|^2\ . \end{aligned} \tag{7}$$

We have calculated this 8-dimensional integral numerically by using the Monte Carlo methods and obtained a simple equation:

$$\sigma_{s.o.} = 2.19 \lambda_C^2 Z_A^2 Z_B^2 \alpha^4 ln^3(\gamma) \tag{8}$$

where 2.19 is the fitted parameter, λ_C is the reduced Compton wavelength of the electron, Z_A and Z_B are the charges of the colliding ions, and γ is the Lorentz factor. In this calculation, we have not included the Coulomb corrections (Fig. 2). In recent articles [12-14], authors consider the Coulomb correction to the electron-positron pair production related to multi-photon exchange of the produced pair with nuclei. This correction to the perturbative cross section is negative and equals −25% at the RHIC and for $Au + Au$ and −14% at the LHC for $Pb + Pb$ collisions. Including the Coulomb cerrections, the total cross section is

$$\sigma_{total} = \sigma_{s.o.} + \sigma_A + \sigma_B + \sigma_{AB} \tag{9}$$

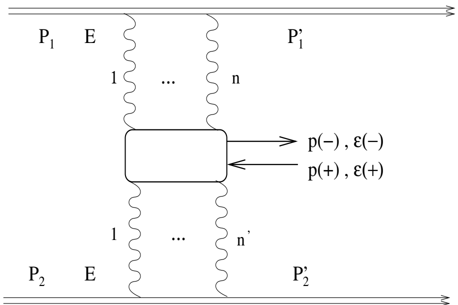

Figure 2. Coulomb correction to the electron-positron pair creation.

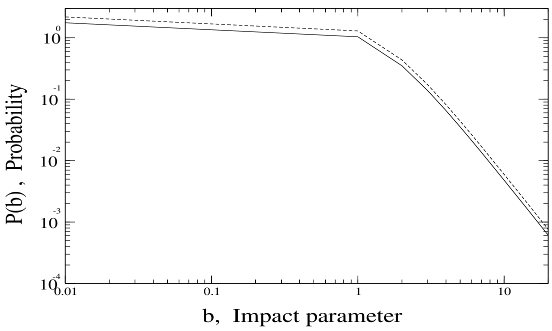

Figure 3. Solid line is the Coulomb corrected result and dashed line is the second order perturbative result. The Coulomb corrections terms reduce the result considerably, however the unitarity is still not restored.

where

$$\sigma_A = -\frac{28(Z_A\alpha)^2(Z_B\alpha)^2}{9\pi m^2} f(Z_A\alpha) ln^2(\gamma^2) \qquad (10)$$

$$\sigma_B = -\frac{28(Z_A\alpha)^2(Z_B\alpha)^2}{9\pi m^2} f(Z_B\alpha) ln^2(\gamma^2) \qquad (11)$$

and

$$\sigma_{AB} = \frac{56(Z_A\alpha)^2(Z_B\alpha)^2}{9\pi m^2} \times f(Z_A\alpha) f(Z_B\alpha) ln(\gamma^2)\,, \qquad (12)$$

where

$$f(Z\alpha) = Z^2\alpha^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2 + Z^2\alpha^2)} \qquad (13)$$

For $Au + Au$ collisions at RHIC energies $f(Z\alpha) = 0.313$. The above detailed expressions can be calculated utilizing Monte Carlo techniques to do the multiple dimensional integrals.

3. Results and Discussion

In the above equations, cross sections are found by integrating over the impact parameter ρ. If we keep the impact parameter in the equations, we can obtain the impact parameter dependence cross section as

$$\frac{d\sigma}{d\rho} = \sigma_{total} \frac{Constant \times \rho}{(Constant^2 + \rho^2)^{3/2}} \qquad (14)$$

where $Constant$ has a value about $1.35\lambda_C$ and comes from the Monte Carlo calculation. From this equation, we can write the probability of the pair production as

$$P(\rho) = \frac{1}{2\pi\rho}\frac{d\sigma}{d\rho} = \sigma_{total} \frac{Constant}{2\pi(Constant^2 + \rho^2)^{3/2}} \qquad (15)$$

As we see from the Fig. 3, this probability is greater than 1 for $\gamma = 100$ and larger. Solid line is the Coulomb corrected result and dashed line is the second order perturbative result. The Coulomb corrections terms reduce the result considerably, however the unitarity is still not restored. From the previous papers, we have showed that including the higher order contributions in the perturbative expansion, this probability can be used as the average number of leptons produced out of the vacuum. Including the higher order contributions, probability can be written as Poisson distribution [15, 16] where $P(\rho)$ is the main function. We have calculated the N pair production cross sections in the Table 1.

In this table, first column shows the produced number of pairs, second column is the probabilities of the corresponding multipair production obtained by using the only second order perturbative result. Third column is the probabilities after Coulomb correction. As we see that, single pair production is the dominant in the collision and multipair production is small, however also measurable. When we include the Coulomb correction terms in the calculations, the results are reduced considerably.

Table 1
Multiple electron-positron pair production probabilities. N is the number of pair produced, second column is the lowest order perturbative result and third column is the Coulomb corrected result.

N	Perturbative (b)	Coulomb Corrected (b)
1	27533	23509
2	4003	3150
3	1293	888
4	451	262

4. Conclusions

In this paper, we have calculated the impact parameter dependence cross section by using the second order perturbative method. We have noticed that for higher energies and large charges unitarity is violated so that probability is greater than one. We have included the Coulomb correction to the second order result. Although this reduce the total cross section significantly, unitarity is still violated. Therefore Coulomb correction alone is not sufficient to restore the unitarity. We need to include the higher order contributions in the perturbative expansion. By making some approximations, we obtained that including all the terms in the expansion gives us a Poisson distribution where the second order term is the main number of the produced leptons.

For the $Au + Au$ case at the RHIC energies, the impact-parameter dependence of the two- and three-pair cross sections are limited to a few Compton wavelengths ($\rho < 10\lambda_C$). We have found that a large portion of the cross section of e^-e^+ pair production resides in the region $\rho > 2R$ ($R \simeq 6.8 fm$ for Au, Compton wavelength for electron $\lambda_C = 386 fm$); thus, we can elliminate much of the concern over hadronic debris for low invariant masses.

5. Acknowledgments

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