ARI The Bulletin of the Istanbul Technical University VOLUME 54, NUMBER 1
Communicated by Derin Orhon

Optimization of Trusses Using the Simulated Annealing Method

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(Received 25 july 2003)

Until very recent times, traditional methods were being used for optimization in structural design. Actually, powered by the advances in computer technology, as to software and hardware, new algorithms have been developed based on heuristic techniques. In these techniques, a large number of solutions are found for the given problem, and choosing the bests among them, a global optimum is searched. Simulated annealing is one of the promising ways to perform this search. In this study the fundamentals of this method are discussed through applications on plane trusses through a special 10-bar truss.

Keywords: Optimization, structural optimization, simulated annealing, two-dimentional trusses, optimum design, optimization method.

1. Introduction

Structural optimization may be defined as development and application of techniques giving best outcome of a given operation while satisfying certain restrictions. Design optimization of trusses aims at arriving at optimum i) topology, ii) configuration, and iii) cross-sectional parameters of members such that the cost or weight of the truss is minimum. Consideration of all the three types of variables makes the problem too complex because of wide variations in the nature of design variables, such as some being discrete and some others being continuous. It is observed that a majority of the methodologies proposed for optimal design of trusses consider only cross-sectional areas of members as design variables. Because of that, to be able to compare the results with existing examples in the literature, only the cross-sectional areas are taken as the design variables and minimization of the weight of the structure as the objective function. A developed program written in Fortran language is used to analyze and optimize the problems with Simulated Annealing (SA) optimization method. Examples are analyzed with stiffness method. Stress limitation of the members and nodal displacements are taken as constraints.

1.1. Simulated Annealing

It is a must to use a computational algorithm to optimize a large-scale problem. For the last forty years researchers have been actively and enthusiastically engaged in the development and investigation of optimality criteria methods, mathematical programming techniques and various computer based algorithms for the solution of optimum structural design problem. Most recently, artificial intelligence methods have gained an important role with very promising results. Methods like, evolutionary strategies, evolutionary programming, genetic algorithms and simulated annealing have been shown to have certain advantages as compared to conventional methods and very successfully used in treating complex and large structures.

The idea of SA originated in statistical mechanics [1], which deal with the equilibrium of large number of atoms in solids and liquids at a given temperature. In the annealing process used in thermodynamics, a physical system (a solid or a liquid) initially at a high-energy state is cooled down to reach the lowest energy state. Idea of that process can be simulated to solve optimization problems by defining a parallelism between minimizing the energy level of a physical system and lowering the objective function [2].

Kirkpatrick *et al.* [3] suggested that this type of simulation could be used to search the feasible solutions of an optimization problem, with the objective of converging to an optimal solution. This approach explores the feasible solutions by repeatedly moving from the current solution to neighboring solution. Energy in Metropolis's approach was replaced by a cost function and the states of a physical system by solutions of a minimization problem.

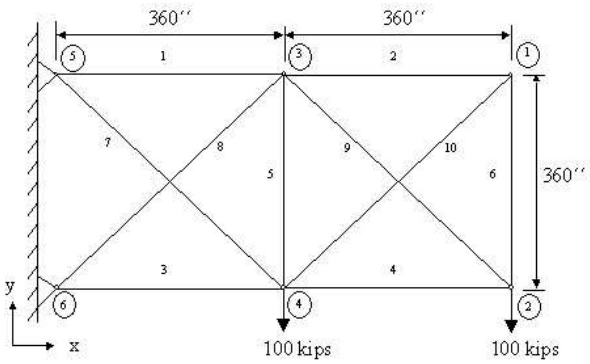

Figure 1. 10-bar truss

Kirkpatrick *et al.* [3] have used traveling salesman problem as a starting point using SA algorithm for the design of computers, where the design variables must be selected from a finite set of discrete values. Balling [4] has developed a SA strategy for use in the discrete optimization of three-dimensional steel frames. Chen, Bruno and Salama [5] have used SA algorithm in optimal placement of active and passive members in complex truss structures. Tagawa and Oshaki [6] represented a continuous topology transition model (CTTM) for plane trusses in general. They have used SA procedures as the optimization technique because traditional gradient-based optimization techniques are difficult to be applied. Moh and Chiang [7] proposed a method on SA that searches from a population as in the method of simulated evolution instead of from a single point. The algorithm is called the region-reduction simulated annealing (RRSA) method because it locates the optimum by eliminating the regions with low probability of containing the optimum. Hasancebi and Erbatur [8] have used SA for layout optimization of trusses.

1.2. The 10-Bar Truss Problem

The 10-bar truss (Fig. 1) is widely used in structural optimization problems as a test example. Numbers of research papers are available in the literature that use classical 10-bar truss to test different optimization techniques.

Rajeev and Krishnamoorthy [9] represent genetic algorithms-based methodologies for obtaining optimal design solutions simultaneously by considering topology, configuration and cross-sectional parameters. The classical 10-bar truss problem is presented to illustrate the size optimization in discrete variables, and apart from this, a topology optimization was presented. Csebfalvis and Csebfalvis [10] have introduced a new branch-and-bound type method in discrete design optimization of geometrically nonlinear truss structures. They have used 10-bar truss to compare their results with sequential and enumeration method present in the literature. Lu et al. [11] have investigated the feasibility of combining neural networks and the gradient methods based algorithms for structural optimization. One of the examples was the 10-bar truss. Guo [12] presented an optimization technique named e-continuation approach, where truss topology optimization problems have been discussed under stress constraints. Vanderplaats and Salajegheh [13] have used the 10-bar truss to demonstrate the efficiency and reliability of a new approximation method for dealing with constraints in structural synthesis. Haftka and Gurdal [14] also have used the 10-bar truss in their studies, which are sequential nonlinear approximate optimization and full stress design (FSD) optimization methods. Full stress design results obtained by Haftka and Gurdal [14], and the new approximation method results found by Vanderplaats and Salajegheh [13] forms the the basis of comparisons with the results obtained from cases in this study.

2. Formulation of the 10-Bar Truss Optimization Problem

The problem can be formulated as a SA problem through the following definitions:

Objective function is a measure of effectiveness of the design. In structural design problems, it is the function to be minimized under given conditions and it may be weight, displacements, stresses, cost or any combination of these.

Design Variables are the parameters in a range that have ability to change or describe the design of the system. They can be cross-sectional dimensions or member sizes, parameters controlling the geometry of the structure, its material properties, etc.

Constraints are conditions that must be met as far as serviceabiltiy and strength of the structure are concerned and those which are imposed because of resource availability. Therefore, as final step during formulating the problem, all the constraints in the system need to be identified.

In general the objective function for the truss problem is the structure weight and denoted by;

$$W = \sum_{i=1}^{N} \rho_i A_i Li \quad (1)$$

where i, is an individual member,
N, total number of members,
ρ_i, material density,
A_i, member cross section area,
and L_i, is the member length.

The design variables may be taken as the member cross-section areas. It is possible to limit them such as;

$$A_i^{min} \leq A_i \leq A_i^{max} \quad (2)$$

where A_i^{min} and A_i^{max} are the minimum and maximum cross-sectional areas. These inequalities then form the first set of constraints of the problem. These constraints may be considered as one type of availability constraints.

Constraints due to serviceability and strength may be defined considering member stresses and nodal displacements:

a. stress constraints

$$\sigma_i^- \leq \sigma_i \leq \sigma_i^+ \quad (3)$$

where σ_i^- is the maximum compression stress for member i, σ_i^+ is the maximum tensile stress for member i.

b. displacement constraints

$$\begin{aligned} X_j^- \leq X_j \leq X_j^+ \\ Y_j^- \leq Y_j \leq Y_j^+ \end{aligned} \quad (4)$$

where X and Y are direction of axis, j is the individual joint, - and + are represents the maximum values in negative and positive directions.

Buckling, of course is another important element in the structural behavior. Buckling constraint is not considered in this study. In fact, inclusion of buckling considerations would not essentially change the procedure.

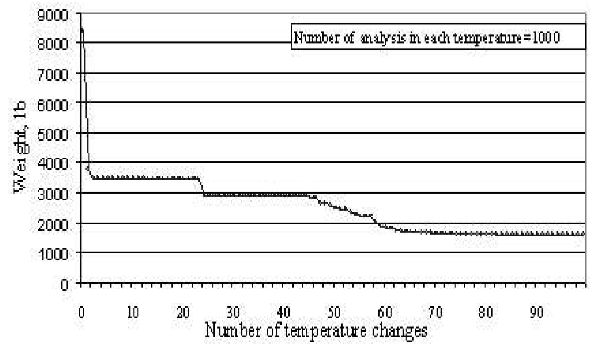

Figure 2. Optimum solution history of 10-bar truss (Case A)

2.1. Description of Algorithm and Program

The algorithm followed is mainly combination of two parts, which are SA and structural analysis. Analysis part is applied as a subroutine to the program where a matrix-stiffness method is used to solve a given truss. All the geometrical and structural data, with the exception of member cross-section areas, are inputs of this part of analysis. Cross-sectional areas, which are design variables are generated by SA algorithm and exchanged between the two parts. Initial cross-sectional areas must be given once to start the algorithm. Constraints such as stress and nodal displacements are considered in such a way that they can be applied separately or together. In SA there are also parameters inherent to the algorithm itself. Initial temperature, temperature reduction value (cooling rate), step size and maximum iteration number are some of these and they are very important to reach the best result with minimum execution time. Initial temperature starts the algorithm acceptance level; the solution is sensitive to the cooling rate, design variables are randomly chosen within the step length and maximum iteration number is a stopping criterion.

This algorithm is programmed in a way to include the following steps:

1. Input all analysis and design information, including initial cross-sectional areas
2. Analyze the truss
3. Evaluate all constraints.

Table 1
Problem data for 10-bar truss

Number of members	10	
Number of design variables	10	
Number of design joints	6	
Applied loads		
Joints	F_x	F_y
2	0	-100 kips
4	0	-100 kips
Young's modulus (E)	10 000 ksi	
Weight density (aluminum)	0.1 lb/in^3	
Allowable stress in tension	25 ksi	
Allowable stress in compression	25 ksi	
Max. displacement in x-direction	2.0 in.	
Max. displacement in y-direction	2.0 in.	
Min. cross-section area	0.1 in^2	
Max. cross-section area	32.0 in^2	

* In Case B allowable stress for member 9 is increased to 50 ksi.

   - If ok then continue, otherwise go to step 6.

4. Check for termination.
   - If maximum iteration is reached then stop, otherwise continue.
5. Calculate weight and check with previous solution.
   - If smaller, then accept as new optimum and continue.
   - If higher, then use Metropolis criteria to decide on acceptance or rejection.
6. Create new areas and go to step 2.

These steps are repeated at each temperature level, starting at a high initial temperature and decreasing by cooling rates, until to reached a global acceptance level.

3. Application on a 10-Bar Truss

The well-known 10-bar truss analyzed many times in the literature is shown in Fig. 1. Table1 represents the data required to carry out analysis and the design. Minimization of the structure weight is taken as the objective function and the member cross- sectional area as the design variables. Every member can have different cross-sectional area, so there are 10 design variables. The material is aluminum with E=10^4 ksi and a specific weight of 0.1lb/in^3.

Three cases are considered in the analysis:

- Case A: All allowable stresses are the same; consider stress constraints only;
- Case B: One member has double allowable stress; consider stress constraints only;
- Case C: All allowable stresses are the same; consider stress constraints and displacement constraints together.

4. Results

All the referenced values and the results are tabulated in Table 2. Column (1) and (2) represents member numbers and initial cross-sectional areas in cubic inches respectively. Column (3) and (5) are the values obtained from Haftka and Vanderplaats implementations with stress constraints only. Column (4) and (6) are the corresponding results obtained from this study. Column (7) and (8) represents Vanderpaats results and results from this study with constraint member stress and joint displacement constraints considered active at the same time.

Fig. 2 shows the optimum solution history of 10-bar truss with weight of truss versus number of temperature changes for Case A. As seen in the figure, initially a rapid reduction occurs in the weight of the structure, followed by a slower reduction with small jumps and slower and very slower reduction at the end, finally implying no reduction any more.

Table 2
Results and comparison of member areas

10-bar truss		Stress constaints only Case A		Case B		Stress+displacement constaints Case C	
Member	initial Area in^2	Ref. 14 Area in^2	This study Area in^2	Ref. 13 Area in^2	This study Area in^2	Ref. 13 Area in^2	This study Area in^2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	20.0	7.34	7.94	7.90	7.93	30.52	30.68
2	20.0	0.10	0.10	0.10	0.10	0.10	0.10
3	20.0	8.06	8.06	8.10	8.10	23.20	23.50
4	20.0	3.94	3.94	3.90	3.90	15.22	14.97
5	20.0	0.10	0.10	0.10	0.10	0.10	0.10
6	20.0	0.10	0.10	0.10	0.10	0.53	0.55
7	20.0	5.74	5.75	5.80	5.80	7.46	7.45
8	20.0	5.57	5.57	5.51	5.53	21.04	21.02
9	20.0	5.57	5.57	3.67	3.68	21.53	21.43
10	20.0	0.10	0.10	0.14	0.14	0.10	0.10
weight,lb	8392.94	1593.2	1593.4	1497.4	1500.3	5060.3	5061.6

Ref.14, implementation with FSD
Case A: Results with only stress constraint [13], Vanderplaats implementation with a new approximation method
Case B: Results referenced with Vanderplaats implementation
Case C: Results referenced from Vanderplaats, addition with displacement constraint.

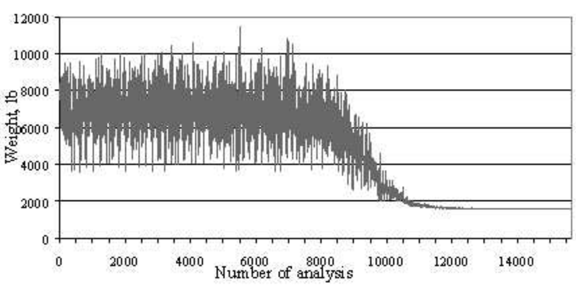

Figure 3. Design history of 10-bar truss (Case A)

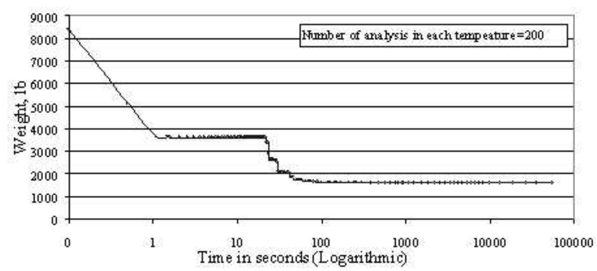

Figure 4. Optimum solution history with respect to time

In Fig. 3 the design history of 10-bar truss shown as a fluctuating curve. Comparisons were made with higher and lower initial temperature. Starting with a higher initial temperature, results in a higher number of iterations at the beginning and better final values, as seen in Fig. 3. Thus better results obtained with increased number of iterations, and consequently with increased execution time. Up and down moves in the curve indicate the normal acceptance points of better values, and rarely worse values in search of global optimum.

Computer applications are performed with a Pentium II-330 processor. Execution times are ranging between 10-15 hours in most cases. As shown in Fig. 4 initially results were obtained very quickly, but as iteration proceeds it requires more time for a small reduction in weight, although an acceptable value was obtained in a short time.

5. Conclusions

By the development and the investigation of new techniques and various computer-based algorithms for the solution of optimum structural problem, metaheuristic methods have gained a great role. One of these techniques is the Simulated Annealing (SA), which is used in this study. Three cases were investigated based on the implementations in the literature and comparisons were carried on with the obtained results. It is observed that SA is effective as other methods in finding the optimum value, although it takes too much time to reach the global optimum because of searching in a wide range of feasible design space and accepting higher values to avoid getting stuck at a local optimum. During this study, a number of cooling rates are tried and the best results are found by to be 0.85. Increasing the initial temperature and iteration number at every temperature level affects the algorithm in a way to yield better results due to searching in a big feasible design region but they increase the execution time. It is concluded that SA is effective as other methods in solving structural optimization problems.

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